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## Optimal power allocation and allowable interference shaping in cognitive radio networks



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#### ABSTRACT

We investigate an interference management problem in multi-carrier cognitive radio systems. To this end, we propose a new framework for protecting primary users such that the primary user can shape the envelope of interference on each subcarrier. Furthermore, we introduce the concept of minimum individual interference budget that the primary user should allow on each subcarrier so that the secondary user can receive a certain quality of service. We develop a joint optimal transmit power and interference budget allocation algorithm for the primary user, which maximizes the throughput of the primary user. Through simulations, we show that our approach can provide better protection for primary users than the existing approach.

#### 1. Introduction

#### 1.1. Background

With the explosive growth of wireless devices, the demand for wireless broadband has been rapidly increasing. To address this issue, a number of communications technologies such as MIMO and OFDM have been developed and adopted in real wireless communication systems. The idea behind those technologies is that higher capacity can be achieved by increasing the dimension of spectrum resources. Recently, an alternative approach, referred to as *cognitive radio*, has emerged as a promising solution for enabling efficient utilization of spectrum resources [1–4]. In cognitive radio systems, unlicensed (or secondary) users are allowed to share the spectrum exclusively allocated to licensed (or primary) users, provided that the secondary user generates limited or zero interference to primary users. In this paper, we focus on the underlay mode where secondary users are allowed to transmit their data simultaneously with primary users [5–10].

#### 1.2. Related work

In this section, we introduce related studies with the problem we will consider in this paper.

#### 1.2.1. Primary protection techniques based on interference power

Clearly, in the underlay mode, a primary receiver can experience interference from secondary senders, and hence, it is necessary to provide a mechanism that can protect primary users (who were to use the spectrum exclusively). One of the well-known spectrum

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underlay methods is to allow the secondary users only if aggregate interference over all subcarriers at the primary receiver is maintained below a certain threshold [11]. While this aggregate interference budget approach enable the secondary user to maximize its system throughput by optimally determining individual interference level on each subcarrier, the primary user is usually excluded in such a determination process, and hence its system throughput is arbitrarily deteriorated.

#### 1.2.2. Primary protection techniques based on achievable rate

More recently, other approaches for primary user protection were proposed by employing more explicit constraints on the performance of primary user (while experiencing interference from secondary user). In [12], the transmit power of secondary user in the single-carrier system is optimized subject to the decodability of primary user's signal, which in turn translates to the minimum rate constraint. On the other hand, [13] considers a multi-carrier system, and imposes the constraint that the total rate loss of primary user due to interference over all subcarriers should be no greater than a certain threshold. Even though these rate-based protection outperforms the conventional ones with interference temperature in terms of the achievable rate of the secondary user, there exist several technical challenges for applying the technique in practical cognitive networks. First, both [12,13] assume global channel state information (CSI), which ultimately requires the information exchange between the primary transmitter and the secondary transmitter. Second, both [12,13] cannot perfectly guarantee the achievable rate of the primary network in *practical* cognitive networks utilizing adaptive modulation and coding scheme (MCS) since unexpected interference for certain sub-carriers may result in packet decoding error. Furthermore, the formulation in [13] is non-convex, making it hard to find an optimal power allocation for secondary user.

#### 1.3. Contributions

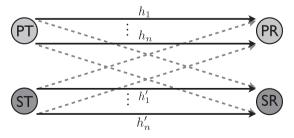
To address the aforementioned problem that primary users can be often subject to unpredictable interference, we propose a new framework for protecting primary users. In particular, our framework allows the primary user to determine individual interference budget on each subcarrier. The individual interference budget is defined as the maximum amount of interference on each subcarrier that the primary user can withstand, and hence the secondary user should not generate interference greater than this budget on each subcarrier. This clearly enables the primary user to achieve guaranteed throughput since interference on each subcarrier can be easily predicted. Furthermore, we introduce the concept of *minimum interference budget* on each subcarrier. Namely, the individual interference budget determined by the primary user should be no smaller than the minimum interference budget, so that the secondary user uses all subcarriers if it wants. We present a mathematical formulation of joint transmit power and individual interference budget allocation for throughput maximization. The formulation is a non-convex optimization problem, which is hard to solve in general. Despite this difficulty, exploiting the structure of the problem, we develop a joint optimal transmit power and individual interference budget allocation algorithm that maximizes the throughput of the primary user subject to minimum interference budget requirements. Note that the proposed algorithm is not optimal in terms of sum-rate of primary and secondary users.

#### 1.4. Organization

The remainder of this paper is organized as follows. Section 2 describes the system model. Section 3 explains the proposed optimization technique. In Section 4, numerical results are shown. Section 5 summarizes the paper with some concluding remarks.

#### 2. System model

We consider the multicarrier cognitive radio system in Fig. 1 where there are one primary user and one secondary user. The set of channel coefficients between primary transmitter (PT) and primary receiver (PR) is  $\{h_i, 1 \le i \le n\}$ , where n is the number of subcarriers. Similarly, over the same set of subcarriers,  $\{h'_i, 1 \le i \le n\}$  denotes the set of channel coefficients between secondary transmitter (ST) and secondary receiver (SR). The channel gain  $a_i$  between PT and PR over each subcarrier i is given as  $a_i = |h_i|^2$ . Similarly,  $a'_i = |h'_i|^2$  denotes the channel gain between ST and SR over each subcarrier i. Let  $p = [p_i, i = 1, ..., n]$  and  $p' = [p'_i, i = 1, ..., n]$  be the transmit power vectors of primary and secondary users, respectively. Also let  $g = [g_i, \forall i]$  and  $g' = [g'_i, \forall i]$ 



**Fig. 1.** The system model of a multicarrier cognitive radio network: solid lines are intended signals and dotted lines are interference. The primary transmitter (PT) transmits to the primary receiver (PR) over n subchannels  $\mathbf{h} = \{h_1, ..., h_n\}$ . The secondary transmitter (ST) transmits to the secondary receiver (SR).

be the channel gain vectors from PT to SR and from ST to PR, respectively. We assume that subcarriers are perfectly orthogonal so that they do not interfere with each other.

We assume the underlay mode of the cognitive radio system where the secondary user is allowed to share the same frequency spectrum with the primary user, provided that it generates limited interference to the primary user. The channel gain vectors g and g' describe interference relationship between primary and secondary users. For instance,  $g_i'p_i'$  is interference from the secondary user to the primary user over subcarrier i. In most of the previous works, the total interference from the secondary user to the primary user is required to stay below a certain level, say Q, which can be expressed as  $\sum_{i=1}^{n} g_i'p_i' \leq Q$ . Under this constraint, however, the primary user can experience unpredictable interference on some subcarriers, which can possibly lead to severe performance degradation in the primary system.

To address this issue, we take an alternative approach that restricts interference on each subcarrier. Specifically, let  $q_i$  be the interference budget on subcarrier i, i.e., the maximum interference that the secondary user is allowed to generate on subcarrier i, which can be written as  $g_i'p_i' \leq q_i$ . We assume that the individual interference budgets  $q_i$ 's are determined by the primary user, subject to the following constraints:

$$q_i \ge \sigma_i(\ge 0), i = 1, ..., n \tag{1}$$

$$\sum_{i=1}^{n} q_i \ge Q,\tag{2}$$

where  $\sigma_i$  is the lower limit on the interference budget on subcarrier i that should be guaranteed for secondary user. The first constraints in (1) require that the individual interference budget over each subcarrier i should be at least  $\sigma_i$ . The second constraint in (2) requires that the total interference budget should be at least Q. This allows the secondary user to share the frequency spectrum with a certain quality while the primary user can control interference limit over each subcarrier. Fig. 2 depicts the individual interference budget  $q_i$  and its lower limit  $\sigma_i$  for every subcarrier. In the next section, we describe how the primary user determines its transmit power vector p and interference budget q. It is assumed that information on q is advertised from the primary user to the secondary user with a periodic or on-demand manner. The secondary user will use the interference budget q to determine its transmit power vector p'. Throughput the paper,  $x = [x_i]$  denotes a vector of length n, where  $x_i$  is the i-th element.

#### 3. Joint optimization of transmit power and interference budget

The goal of the primary system is to maximize the system throughput subject to the requirements  $\sigma = [\sigma_i, \forall i]$  and Q on the allowable interference limit  $q = [q_i, \forall i]$ . This problem can be formulated as follows:

(MaxRate) 
$$\max_{p,q} \sum_{i=1}^{n} \log \left( 1 + \frac{p_i a_i}{q_i + N_0} \right)$$
 (3)

subject to 
$$p \ge 0$$
,  $\sum_{i=1}^{n} p_i \le P_{\text{max}}$  (4)

$$q \succeq \sigma, \ \sum_{i=1}^{n} q_i \ge Q,$$
 (5)

where we use " $\geq$ " to denote element-wise inequality for vectors, and  $N_0$  is the noise power over each subcarrier. We assume that every subcarrier is subject to the same noise power  $N_0$ . The objective function (3) is the total throughput of the primary system where each subcarrier i is assumed to experience interference  $q_i$  from the secondary system. The constraints in (4) represent the nonnegativity of p and the peak power limit. The constraints in (5) require that the allowable interference at each subcarrier i should be no smaller than  $\sigma_i$  and the total allowable interference should be no smaller than Q. Hence, the optimal solution to the above problem gives a primary power allocation and allowable interference limit over each subcarrier that maximizes the primary system

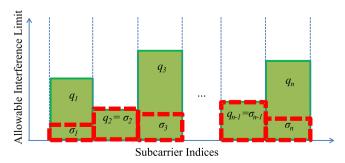


Fig. 2. An example of the individual interference budget for n subcarriers. Each interference budget  $q_i$  should be at least  $\sigma_i$ . Also, the total interference budget  $\sum_{i=1}^{n} q_i$  should be at least Q.

throughput.

#### 3.1. Optimal power allocation and interference shaping algorithm of primary user

Clearly, the problem is nonconvex, which is hard to solve in general. However, we use the structure of the problem to develop an efficient optimal algorithm. Our algorithm is based on the following well-known fact about convex maximization problems [14, p.134]:

**Lemma 1.** Let  $f: R^n \to R$  be a convex function, and S be a nonempty compact polyhedral set in  $R^n$ . Then, an extreme point of S is an optimal solution to the problem of maximizing f(x) subject to  $x \in S$ .

Using this lemma, we can prove the following:

**Theorem 1.** Let  $F = \{q: q \succeq \sigma, \sum_{i=1}^{n} q_i = Q\}$ . Then, the optimal solution to the problem (MaxRate) exists at a point where q is an extreme point of F.

**Proof.** First, note that an optimal solution exists at a point satisfying  $\sum_i q_i = Q$  since any  $\mathbf{q}$  with  $\Sigma_i q_i > Q$  yields no higher throughput. Hence, the constraint  $\Sigma_i q_i \geq Q$  can be replaced by  $\sum_i q_i = Q$ . That is, (5) can be replaced by F. Clearly, F is a compact polyhedral set. Let  $(p^*, q^*)$  be an optimal solution to the problem (MaxRate). Consider the problem (MaxRate) with fixed  $p = p^*$ , which is written as

$$\max_{q \in F} \quad \sum_{i=1}^{n} \log_2 \left( 1 + \frac{p_i^* a_i}{q_i + N_0} \right) \tag{6}$$

The above problem is a convex maximization over a compact polyhedral set F, and thus, an extreme point of F is an optimal solution. Furthermore, the optimal objective function value of the reduced problem is equal to that of the original problem (MaxRate) (i.e., the objective function value at  $(p^*, q^*)$ ). Therefore,  $(p^*, \tilde{q})$  is also an optimal solution to the original problem where  $\tilde{q}$  is an optimal extreme point solution to the reduced problem. This shows that the optimal solution of the problem (MaxRate) is attained at an extreme point of F.

Note that F has total n extreme points, which we denote by  $\{x^1, x^2, ..., x^n\}$ . Here, each  $x^j$  denotes a vector  $q \in R^n$  such that  $q_j = Q - \sum_{i \neq j} \sigma_i$  and  $q_i = \sigma_i$ ,  $\forall i \neq j$ . By Theorem 1, one of these n points is an optimal  $q^*$ . As a consequence, solving the problem (MaxRate) with  $q = x^j$  for each j, and taking a solution with maximum throughput give an optimal solution  $(p^*, q^*)$ . Based on this observation, we propose the following algorithm where  $p^j$  is an optimal solution to the problem (MaxRate) with  $q = x^j$  and R(p, q) denotes the sum rate under (p, q).

Note that the problem in step (2).(a) can be solved in polynomial time using the well-known waterfilling algorithm. Therefore, our algorithm can find an optimal power allocation and individual interference budget in polynomial time.

Note also that some subcarriers may not be used by the primary user, as shown in the following lemma:

Lemma 2. Suppose that for each subcarrier j, there exists a subcarrier k (depending on j) such that

$$P_{\max} + \sum_{i \neq k} \frac{x_i^j + N_0}{a_i} \le \frac{(n-1)(x_k^j + N_0)}{a_k}.$$
 (7)

Then, under the optimal solution of (MaxRate), there exists at least one subcarrier which is not used by the primary system.

**Proof.** Let  $(p^*, q^*)$  be an optimal solution to the problem (MaxRate). By Theorem 1, there is a subcarrier, say j, such that  $q^* = x^j$ . Let  $[\cdot]_m^M$  denote min  $\{M, \max\{\cdot, m\}\}$ . It is easy to see (from the well-known waterfilling algorithm) that the optimal  $p^*$  is given by

$$p_i^* = \left[\frac{1}{\lambda} - \frac{x_i^j + N_0}{a_i}\right]_0^\infty, \,\forall i,$$
(8)

where  $\lambda$  is a positive number such that  $\sum_i p_i^* = P_{\text{max}}$ . Now, assume that every subcarrier is used by the primary system, i.e.,

$$\frac{1}{\lambda} > \frac{x_i^j + N_0}{a_i}, \forall i$$
 (9)

The optimal  $p^*$  is then written as

$$p_i^* = \frac{1}{\lambda} - \frac{x_i^j + N_0}{a_i}, \, \forall i,$$
 (10)

and thus the value of  $\lambda$  can be obtained as

$$\lambda = \frac{n}{P_{\text{max}} + \sum_{i} \frac{x_{i}^{j} + N_{0}}{a_{i}}}.$$
(11)

Plugging this back into (9) yields

$$\frac{P_{\max}}{n} + \frac{1}{n} \sum_{i} \frac{x_i^{j} + N_0}{a_i} > \frac{x_i^{j} + N_0}{a_i}, \forall i$$
 (12)

This contradicts the assumption (7), and therefore, not every subcarrier is used.  $\Box$ 

Note that the condition in (7) is satisfied, for instance, when  $P_{\text{max}}$  is small. Clearly, the secondary user can allocate its transmit power to the subcarriers that are not used by the primary user, without affecting the performance of primary user. Therefore, we assume that after running the optimal algorithm, the primary user updates  $q^*$  such that  $q_i^* = \infty$  if  $p_i^* = 0$ . This will allow the secondary user to further enhance its performance. The optimal solution of MaxRate can give an aggregated interference budget allocated for each subcarrier, exceeding Q. However, this case happens only when there is at least one subcarrier which is not used by the primary user. The performance of primary user is not affected no matter how much power the secondary user allocates to such a subcarrier, provided that subcarriers are orthogonal. Note that if every subcarrier is used by the primary user, an interference budget allocation exceeding Q is clearly not an optimal solution of MaxRate.

#### 3.2. Power allocation policy of secondary user

The secondary user receives the interference budget vector  $q^*$  from the primary user, and uses the information when computing its power allocation over subcarriers. Recall that for secondary user, we use the same notation as in the primary user, except with '. For instance,  $a_i'$  denotes the channel gain of secondary user over subcarrier i. Obviously, the secondary user would desire to maximize its throughput, and hence the secondary user's power allocation problem can be formulated as follows:

$$\max_{p' \ge 0} \quad \sum_{i=1}^{n} \log \left( 1 + \frac{a_i' p_i'}{g_i p_i^* + N_0} \right) \tag{13}$$

subject to 
$$\sum_{i=1}^{n} p_i' \le P_{\text{max}}'$$
 (14)

$$g_i' p_i' \le q_i^*, \forall i \tag{15}$$

where  $g_i$  (or  $g_i'$ ) is the channel gain from primary (or secondary) user's transmitter to secondary (or primary) user's receiver. The constraints in (15) require that for each subcarrier, the interference from the secondary user to the primary user should not exceed the limit allowed by the primary user.

The above problem is a convex optimization, and its optimal solution can be obtained using the constrained waterfilling algorithm as follows [11]:

$$p_i' = \left[\frac{1}{\theta} - \frac{g_i p_i^* + N_0}{a_i'}\right]_0^{\frac{q_i^*}{g_i'}}, \, \forall i,$$
(16)

where  $\theta$  is the smallest positive constant such that  $\sum_i p_i' \leq P_{\max}'$ . Note that the optimal power vector p' may not fully utilize the interference budget  $q^*$ , i.e., there could exist a subcarrier i such that  $g_i'p_i' < q_i^*$  and  $q_i^* < \infty$ . Consequently, the actual throughput of primary user can be possibly greater than the original throughput computed through the problem (MaxRate).

In the next section, we will compare our policy with the policy in [11], which only limits the *aggregate* interference that the secondary user can generate (we call this policy the AIB (Aggregate Interference Budget) policy). That is, the secondary user's transmit power should satisfy

$$\sum_{i} g_i' p_i' \le Q. \tag{17}$$

Clearly, this constraint is less restrictive than the constraints in (15), and thus the secondary user will be able to achieve higher throughput than under the individual constraints on interference (15). However, on each subcarrier, the primary user can experience unpredictable interference from the secondary user. On the other hand, in our policy, the primary user advertises the tolerable interference on each subcarrier, so that it can predict interference level from the secondary user.

It should be noted that the AIB policy does not specify the power allocation of the primary user, but studies only the secondary user's power allocation algorithm. However, for comparison, we assume that the same primary transmit power  $p^*$  is used for the primary user under the AIB policy.

#### 4. Simulation results

We present some numerical results to examine the performance of our algorithm. There are 20 subcarriers, and the peak power of each user is 5, i.e.,  $P_{\text{max}} = P'_{\text{max}} = 5$ . The noise power is 1/n. The value of Q is fixed to 1. Each channel coefficient  $h_i$  follows the standard normal distribution [15], and they are assumed to be independent across subcarriers and users. The lower limit  $\sigma_i$  on interference budget of each subcarrier i is set to  $\alpha/n$  and the value of  $\alpha$  is varied from 0 to 1. Our algorithm is compared with the following algorithms in the literature.

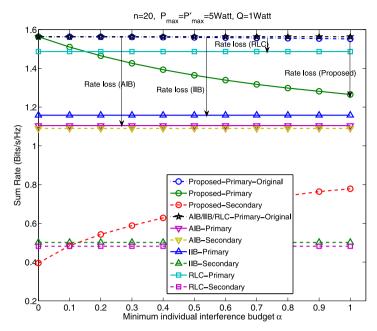


Fig. 3. Throughput of primary and secondary users under each policy: Each point in the plot is the average of 5000 realizations of the channels.

- Aggregate Interference Budget (AIB) policy [11] that requires total interference from SU to PU to stay below a certain threshold (see (17)). The threshold *Q* is fixed to 1.
- Individual Interference Budget (IIB) policy that requires interference from SU to PU on each subcarrier to stay below a certain threshold. The threshold is fixed to 1/n for each subcarrier. Thus, the maximum possible interference from SU to PU is 1, which is equal to the aggregate interference threshold Q in AIB policy.
- Rate Loss Constraint (RLC) policy [13] that requires the rate loss of PU to be no greater than a certain value. The rate loss constraint is converted into minimum rate constraint [13], and in simulations, the minimum rate (or spectral efficiency) for PU is set to 1Bits/s/Hz.

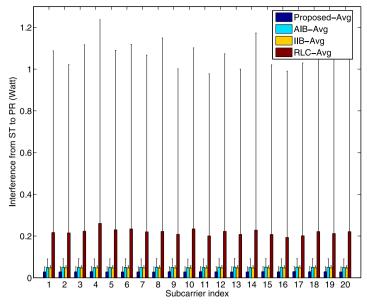
The power allocation of primary user for the three policies is determined via water-filling algorithm, assuming no interference from secondary user.

Fig. 3 shows the sum rate of primary and secondary users. It also shows the original sum rate that the primary user would achieve without secondary user. Hence, the original sum rate minus the achieved sum rate is the rate loss of primary user. Note that the sum rates of the three policies remain unchanged (as the value of  $\alpha$  changes) because the threshold parameters in the three policies are all fixed. On the other hand, under our policy, as the minimum individual interference budget  $\alpha$  increases, the sum rate of primary user decreases whereas that of secondary user increases. This is because the secondary user is granted higher power budget on each subcarrier as  $\alpha$  increases.

Note also that the sum rate of primary user is larger under our policy than under AIB policy. In contrast, the AIB policy achieves higher secondary sum rate because the secondary user has a larger feasibility region for its transmit power under the aggregate interference budget constraint. IIB policy also achieves higher primary sum rate than AIB policy because it controls individual interference on each subcarrier. Similar to comparison of AIB and our policies, AIB policy achieves higher secondary sum rate than IIB since it gives larger feasible region for secondary to optimize over.

RLC policy guarantees higher sum rate for primary user than all the other policies, but the secondary user achieves the lowest sum rate. In other words, the rate loss is minimum under RLC policy, showing that the primary user is better protected than under any other policy. When  $\alpha=1$ , our policy as well as AIB and IIB constrains the aggregate interference below 1. In this case, the rate loss is smaller under our policy than under AIB and IIB policies. Moreover, our policy achieves higher secondary sum rate than IIB policy. This is because under our policy, the primary user maximizes its sum rate knowing the maximum possible interference on each subcarrier whereas AIB and IIB policies just maximize the sum rate assuming no interference. Furthermore, under our policy, the secondary user uses the information on which subcarriers are not used by primary user; so that it can freely use those subcarriers without harming PU.

Fig. 4 shows the average and standard deviation of interference from the secondary transmitter to the primary receiver. We assumed that interference is zero on any subcarrier unused by primary user. Our policy and IIB policy control individual interference so that interference from SU to PU is kept smaller than a threshold. Under AIB policy and RLC policy, interference on each subcarrier can be vastly different depending on channel conditions, and hence, it is difficult for the primary user to predict and control the performance over each subcarrier. RLC policy experiences much higher average interference (and standard deviation) than other policies. We found that the RLC policy tends to find a secondary user power allocation such that only a few subcarriers are used. As a



**Fig. 4.** Average and standard deviation of (5000 channel realizations) individual interference from ST to PR: n = 20,  $\sigma_i = 1/n$ ,  $\forall i, Q = 1$ . Both of our algorithm and AIB algorithm use the same value of Q. Interference over subcarriers unused by primary user is assumed to be zero.

consequence, the secondary user generates excessively high interference on a small number of subcarriers for each channel realization<sup>1</sup>. This is why the average interference (and variance as well) is high under RLC policy.

#### 5. Conclusion

In this paper, we studied the problem of protecting primary users in multicarrier cognitive radio systems. The focus of this study has been on secondary user's throughput maximization under aggregate interference budget constraint. We proposed a new framework in which the primary user jointly optimizes its transmit power and individual interference budget on each subcarrier subject to minimum interference budget requirements. Our framework enables the primary user to better predict its performance while the secondary user is guaranteed a certain level of quality of service. We showed through simulations that our approach provides better protection for primary user than the existing approach.

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#### Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.compeleceng.2018.07.015

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<sup>&</sup>lt;sup>1</sup> A plausible explanation is that the formulation of RLC policy involves non-convex constraints and thus the optimal solution is likely to exist near an extreme point

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